

Neutrino oscillations and rare processes in models with a small extra dimension

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Abstract

We discuss Dirac neutrino masses and mixings in a scenario where both the standard model fermions and right handed neutrinos are bulk fields in a non-factorizable geometry in five dimensions. We show how the atmospheric and solar neutrino anomalies can be satisfactorily resolved, and in particular how bimaximal mixing is realized. We also consider rare processes such as neutron-antineutron oscillations and $\mu \rightarrow e + \gamma$, which may occur at an observable rate.

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1 Introduction

The huge discrepancy between the Planck scale $M_{\text{Pl}} \sim 10^{19}$ GeV and the scale of electroweak symmetry breaking $M_Z \sim 10^2$ GeV, is one of the most interesting challenges in modern physics. Recently it was realized that compact extra space dimensions can offer a new perspective on this gauge hierarchy problem. In one class of models the weakness of 4-dimensional gravity is induced by the very large volume of compactification [1]. Subsequently, it was demonstrated that a small but warped extra dimension provides an elegant alternative solution to the hierarchy problem [2] (see also [3]). The fifth dimension is an S_1/Z_2 orbifold with an AdS_5 geometry, bordered by two 3-branes with opposite tensions and separated by distance R . The AdS warp factor $\Omega = e^{-\pi k R}$ generates an exponential hierarchy between the effective mass scales on the two branes ($k=\text{AdS curvature}$). If the brane separation is $kR \simeq 11$, the scale on the negative tension brane is of TeV-size, while the scale on the other brane is of order M_{Pl} . The AdS curvature k and the 5d Planck mass M_5 are both assumed to be of order M_{Pl} . At the TeV-brane gravity is weak because the zero mode corresponding to the 4d graviton is localized at the positive tension brane (Planck-brane).

In low scale models of quantum gravity higher-dimensional operators, now only TeV-scale suppressed, are known to induce rapid proton decay, large neutrino masses and flavor violating interactions. In models with non-factorizable (warped) geometry these problems can be cured to some extent by introducing the Standard Model (SM) fermions as bulk fields, without relying on ad-hoc symmetries such as lepton or baryon number [4, 5]. Because of the warp factor, the effective cut-off scale varies along the extra dimension. If the quarks and leptons are localized towards the Planck-brane in the extra dimension, the effective cut-off scale can be much larger than a TeV.

Fermion masses are generated by the Higgs mechanism. In the non-supersymmetric framework we are discussing, the Higgs field lives close to the TeV-brane in order to maintain the solution of the gauge hierarchy problem [6–8]. The induced fermion masses crucially depend on the overlap between the Higgs and fermion wave functions in the extra dimension, and naturally become small for fermions residing close to the Planck-brane. This mechanism offers a higher-dimensional view on the problem of fermion mass hierarchies [4]. Also, the quark mixings can be nicely reproduced from order unity Yukawa couplings [5]. The nearest neighbor structure in the CKM matrix is reflected in the different positions the fermions occupy in the extra dimension.

In our set-up we assume that the SM gauge bosons are bulk fields as well in order to guarantee bulk gauge invariance. Electroweak precision data [4, 6, 8, 9], especially the W and Z boson mass ratio, impose stringent constraints by requiring the Kaluza Klein (KK) excitations of bulk gauge bosons and fermions to have masses of order 10 TeV [7]. The measured W and Z boson masses are reproduced by some (mild) tuning of parameters. We note in passing that contributions of 10 TeV Kaluza-Klein

states to the anomalous magnetic moment of the muon are two to three orders of magnitude smaller [10] than the recently reported deviation of this observable from its SM prediction [11].

Taking into account the constraints on the locations of fermions from their masses and mixings, one can more reliably estimate the suppression scales for higher-dimensional operators. Even though flavor violating interactions are safely suppressed, four-fermion operators suppressed by mass scales of about 10^{12} GeV can still lead to proton decay at an unacceptable level [5], unless the dimensionless couplings happen to be $\lesssim 10^{-7}$. In addition keV-size Majorana neutrino masses and small mixings are induced, which do not easily fit the neutrino oscillation data [12].

In this letter we study the generation of Dirac masses for the neutrinos by introducing right-handed (sterile) neutrinos in the bulk. In the case of TeV-brane SM neutrinos this scenario was put forward in ref. [13]. It turns out that we can accommodate the atmospheric and solar neutrino oscillations without fine-tuning of parameters. Having the SM neutrinos in the bulk reduces their mixing with Kaluza-Klein sterile neutrinos. Lepton flavor violating processes, which in the scenario of ref. [13] are in conflict with experiment for Kaluza Klein masses below 25 TeV [14], are safely suppressed. In this respect, moving the SM fermions off the TeV-brane allows for smaller KK masses ~ 10 TeV and therefore reduces the amount of fine-tuning to obtain realistic weak boson masses. The unacceptably large Majorana neutrino masses (see previous paragraph) are eliminated by imposing lepton number symmetry. Another important consequence is that the proton becomes stable. However, certain baryon number violating processes, e.g. neutron anti-neutron oscillations with $\Delta B = 2$, are still possible and may lead to interesting experimental signatures [15, 16].

2 Bulk fermions

To set the notation let us briefly review some properties of fermions in a slice of AdS_5 . We consider the non-factorizable metric [2]

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (2.1)$$

where $\sigma(y) = k|y|$. The 4-dimensional metric is $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, k is the AdS curvature related to the bulk cosmological constant and brane tensions, and y denotes the fifth coordinate. The equation of motion for a fermion in curved space-time reads

$$E_a^M \gamma^a (\partial_M + \omega_M) \Psi + m_\Psi \Psi = 0, \quad (2.2)$$

where E_a^M is the fünfbein, $\gamma^a = (\gamma^\mu, \gamma^5)$ are the Dirac matrices in flat space,

$$\omega_M = \left(\frac{1}{2} e^{-\sigma} \sigma' \gamma^5 \gamma_\mu, 0 \right) \quad (2.3)$$

is the spin connection, and $\sigma' = d\sigma/dy$. The index M refers to objects in 5d curved space, the index a to those in tangent space. Fermions have two possible transformation properties under the Z_2 orbifold symmetry, $\Psi(-y)_\pm = \pm\gamma_5\Psi(y)_\pm$. Thus, $\bar{\Psi}_\pm\Psi_\pm$ is odd under Z_2 , and the Dirac mass term, which is also odd, can be parametrized as $m_\Psi = c\sigma'$. The Dirac mass should therefore originate from the coupling to a Z_2 odd scalar field which acquires a vev. On the other hand, $\bar{\Psi}_\pm\Psi_\mp$ is even. Using the metric (2.1) one obtains for the left- and right-handed components of the Dirac spinor [4, 13]

$$[e^{2\sigma}\partial_\mu\partial^\mu + \partial_5^2 - \sigma'\partial_5 - M^2]e^{-2\sigma}\Psi_{L,R} = 0, \quad (2.4)$$

where $M^2 = c(c \pm 1)k^2 \mp c\sigma''$ and $\Psi_{L,R} = \pm\gamma_5\Psi_{L,R}$.

Decomposing the 5d fields as

$$\Psi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Psi^{(n)}(x^\mu) f_n(y), \quad (2.5)$$

one ends up with a zero mode wave function [4, 13]

$$f_0(y) = \frac{e^{(2-c)\sigma}}{N_0}, \quad (2.6)$$

and a tower of KK excited states

$$f_n(y) = \frac{e^{5\sigma/2}}{N_n} \left[J_\alpha\left(\frac{m_n}{k}e^\sigma\right) + b_\alpha(m_n)Y_\alpha\left(\frac{m_n}{k}e^\sigma\right) \right]. \quad (2.7)$$

The order of the Bessel functions is $\alpha = |c \pm 1/2|$ for $\Psi_{L,R}$. The spectrum of KK masses m_n and the coefficients b_α are determined by the boundary conditions of the wave functions at the branes. The normalization constants follow from

$$\frac{1}{2\pi R} \int_{\pi R}^{\pi R} dy e^{-3\sigma} f_m(y) f_n(y) = \delta_{mn}. \quad (2.8)$$

Because of the orbifold symmetry, the zero mode of Ψ_+ (Ψ_-) is a left-handed (right-handed) Weyl spinor. For $c > 1/2$ ($c < 1/2$) the fermion is localized near the boundary at $y = 0$ ($y = \pi R$), i.e. at the Planck- (TeV-) brane.

The zero modes of leptons and quarks acquire masses from their coupling to the Higgs field

$$\int d^4x \int dy \sqrt{-g} \lambda_{ij}^{(5)} H \bar{\Psi}_{i+} \Psi_{j-} \equiv \int d^4x m_{ij} \bar{\Psi}_{iR}^{(0)} \Psi_{jL}^{(0)} + \dots, \quad (2.9)$$

where $\lambda_{ij}^{(5)}$ are the 5d Yukawa couplings. The 4d Dirac masses are given by

$$m_{ij} = \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} \lambda_{ij}^{(5)} e^{-4\sigma} H(y) f_{0iL}(y) f_{0jR}(y). \quad (2.10)$$

Recall that the Higgs field is confined to the TeV-brane, i.e. $H(y) = H_0\delta(y - \pi R)$. Using the known mass of the W-boson we can fix H_0 in terms of the 5d weak gauge coupling $g^{(5)}$.

The fermion wave functions and consequently their masses depend on the 5d mass parameters of the left- and right-handed fermions, c_L and c_R respectively, which enter (2.10). As the 5d Dirac mass, i.e. c parameter of the fermion increases, the closer it gets closer localized towards the Planck-brane. Its overlap with the Higgs profile at the TeV-brane is consequently reduced, which is reflected in a smaller 4d fermion mass after electroweak symmetry breaking. In ref. [5] we have shown that this geometrical picture beautifully generates the charged lepton and quark mass hierarchies, as well as quark mixings, by employing c -parameters and $\lambda_{ij}^{(5)}\sqrt{k}$ of order unity. In the following we apply this mechanism to also generate small neutrino masses consistent with neutrino oscillation experiments.

3 Neutrino masses

In ref. [13] it was suggested that in warped geometry models small Dirac neutrino masses arise from a coupling to sterile (right-handed) bulk neutrinos. In order to generate masses in the sub-eV range, the sterile neutrinos have to be localized close to the Planck-brane, while the SM model neutrinos were confined to the TeV-brane. Here we generalize the scenario to incorporate bulk SM neutrinos.

In our framework the SM fermions, described by 4d Weyl spinors, correspond to left- and right-handed zero modes of 5d Dirac spinors which live in the bulk. The 5d mass parameters of the $SU(2)$ doublet (singlet) leptons L_i (E_i) are $c_L^{(i)}$ ($c_E^{(i)}$), where $i = 1, 2, 3$. The right-handed neutrinos ψ_i are also associated with bulk fermion fields which have right-handed zero modes. Their 5d mass parameters are denoted by c_ψ^i , where again $i = 1, 2, 3$. We avoid large Majorana neutrino masses by imposing lepton number as discussed earlier. After electroweak symmetry breaking the Dirac masses for the neutrinos are generated from the Yukawa-type coupling between the SM and sterile neutrinos, analogous to eq. (2.9)

$$\mathcal{L} = h_{ij}^{(5)} \bar{L}_i \psi_j H + \text{h.c.} + \dots \quad (3.11)$$

We will demonstrate that the neutrino oscillation data can be reproduced with order unity Yukawa couplings $h_{ij}^{(5)}$.

From the KK reduction of the left-handed neutrino fields ν_L we obtain a left-handed zero mode $\nu_L^{(0)}$, corresponding to the SM neutrinos, and an infinite tower of left- and right-handed KK excited states $\nu_L^{(i)}$ and $\nu_R^{(i)}$, where we omit flavor indices. The sterile neutrinos decompose into the right-handed zero mode $\psi_R^{(0)}$ and the KK excited states $\psi_L^{(i)}$ and $\psi_R^{(i)}$. After electroweak symmetry breaking the mass matrix

takes the form

$$M_\nu = (\nu_L^{(0)}, \nu_L^{(1)}, \psi_L^{(1)}, \dots) \begin{pmatrix} m^{(0,0)} & 0 & m^{(0,1)} & \dots \\ m^{(1,0)} & m_{L,1} & m^{(1,1)} & \dots \\ 0 & 0 & m_{\psi,1} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \psi_R^{(0)} \\ \nu_R^{(1)} \\ \psi_R^{(1)} \\ \vdots \end{pmatrix} \quad (3.12)$$

where we again suppress flavor indices, i.e. every entry represents a 3×3 matrix in flavor space. The masses $m^{(i,j)}$ are obtained by inserting the relevant wave functions into eq. (2.10). The $m_{L,i}$ and $m_{\psi,i}$ are the KK masses of the excited neutrino states. The zeros in (3.12) follow from the Z_2 orbifold properties of the wave functions. In ref. [7] we used an exponential Higgs profile localized at the TeV-brane instead of strictly confining it to the brane. In that case a mass term arises for $\psi_L^{(1)} \nu_R^{(1)}$, etc. We have checked that including this term only affects the neutrino properties at the 10^{-3} level (at most).

The squares of the physical neutrino masses are the eigenvalues of the hermitian mass matrix $M_\nu M_\nu^\dagger$. The unitary matrix U , such that $U^\dagger M_\nu M_\nu^\dagger U$ is diagonal, relates the left-handed mass eigenstates N_L^{phys} to the interaction eigenstates $N_L = (\nu_L^{(0)}, \nu_L^{(i)}, \psi_L^{(i)})$ via $N_L = U N_L^{\text{phys}}$. The physical neutrinos ν_L^{phys} correspond to the three lightest states in N_L^{phys} . The right-handed mass eigenstates are obtained from a unitary matrix V which diagonalizes $M_\nu^\dagger M_\nu$. If the Yukawa couplings of the charged leptons are diagonal, the mixing angles of the physical neutrinos can be directly read off from U .

The atmospheric and solar neutrino anomalies can be solved by assuming oscillations among the different neutrino flavors via the mass matrix M_ν (see e.g. [12] for a recent review). The atmospheric neutrino flux is reproduced by $\Delta m_{\text{atm}}^2 = 2\text{--}7 \cdot 10^{-3}$ eV² and an almost maximal mixing angle $\sin^2 2\theta_{\text{atm}} \sim 1$. For the solar neutrinos a variety of masses and mixing angles fit the data. For the large mixing angle MSW solution one finds $\Delta m_{\text{sol}}^2 = 10^{-5}\text{--}10^{-3}$ eV² and $\sin^2 2\theta_{\text{sol}} \sim 1$, while for the small mixing angle MSW solution $\Delta m_{\text{sol}}^2 = 5 \cdot 10^{-6}\text{--}10^{-5}$ eV² and $\sin^2 2\theta_{\text{sol}} = 10^{-3}\text{--}10^{-2}$. Similarly, for the LOW solution $\Delta m_{\text{sol}}^2 = 5 \cdot 10^{-8}\text{--}10^{-7}$ eV² and $\sin^2 2\theta_{\text{sol}} \sim 1$ and for the vacuum solution $\Delta m_{\text{sol}}^2 \sim 10^{-10}$ eV² and $\sin^2 2\theta_{\text{sol}} \sim 1$. Finally, the CHOOZ reactor experiment together with the atmospheric neutrino data constrain $|U_{e3}|^2$ to be smaller than a few times 10^{-2} . In the next section we demonstrate how neutrino mass matrices with these properties can be obtained from bulk neutrinos.

Our approach to neutrino oscillations is in the spirit of “neutrino mass anarchy” models [17]. It has been demonstrated that a fair fraction of neutrino mass matrices, which appear to have random entries, are consistent with the experimental constraints. This works best for the large MSW solution to the solar neutrino anomaly, where the ratio of $\Delta m_{\text{atm}}^2 / \Delta m_{\text{sol}}^2$ is not too large, and no tiny mixing angles are involved. In our analysis of neutrino oscillations we assume non-hierarchical Yukawa couplings $h_{ij}^{(5)}$ on order of the 5d gauge coupling $g_2^{(5)}$. However, in our framework the structure of the Yukawa couplings is not mapped one to one to a neutrino

mass matrix. Rather, the Yukawa texture is deformed by the neutrino wave function in the extra dimension which enter eq. (2.10). Therefore, it is not trivial that the neutrino data can be obtained with order unity Yukawa couplings. On the other hand, one may hope that the wave functions induce hierarchies that allows one to also reproduce also the small angle MSW, LOW and vacuum solution to the solar neutrino problem.

4 Numerical results

Various constraints on the scenario with bulk gauge and fermion fields have been discussed in the literature [4, 6–8, 10, 22]. With bulk gauge fields for instance, the SM relationship between the gauge couplings and masses of the Z and W bosons gets modified. The electroweak precision data then requires the lowest KK excitation of the gauge bosons to be heavier than about 10 TeV [7]. This bound becomes especially important if the fermions are localized towards the Planck-brane ($c > 1/2$). As discussed in ref. [5], this applies to all SM fermions, with the exception of the top-quark, if the fermion mass hierarchy arises from their different locations in the extra dimension. In this case the bounds induced by the contribution of KK excitations of the SM gauge bosons to the electroweak precision observables are weak. They only require the KK masses to be above about 1 TeV [4, 8]. In the following we will therefore assume that the mass of the first KK gauge boson is $m_1^{(G)} = 10$ TeV. The corresponding masses of the KK fermions are then in the range 10 to 16 TeV, for $0 < c < 1.5$. The mass of the lightest KK graviton is 16 TeV.

For simplicity we assume diagonal Yukawa couplings $\lambda_{ij}^{(5)}$ for the charged leptons. Neutrino mixing is then solely governed by the neutrino mass matrix M_ν . To avoid a hierarchy in the 5d couplings, we assume $\lambda_{ii}^{(5)} = g_2^{(5)}$, where the 5d weak gauge coupling $g_2^{(5)} \sim g_2 \sqrt{2\pi R}$. We take $k = M_5 = \overline{M}_{\text{Pl}}$, where $\overline{M}_{\text{Pl}} = 2.44 \times 10^{18}$ GeV is the reduced Planck mass. From $m_1^{(G)} = 10$ TeV we determine the brane separation $kR = 10.83$ [9].

As discussed in ref. [5] the measured charged lepton masses do not determine the 5d mass parameters $c_L^{(i)}$ and $c_E^{(i)}$ in a unique way. In order to minimize Majorana neutrino masses, we proposed a scenario where the left-handed leptons L_i were placed as close as possible towards the Planck-brane, while the right-handed leptons E_i were assumed to be delocalized. The associated fermion mass parameters are [5]

$$c_L^{(1)} = 0.834, \quad c_L^{(2)} = 0.664, \quad c_L^{(3)} = 0.567, \quad c_E^{(1)} = c_E^{(2)} = c_E^{(3)} = 0.5. \quad (4.13)$$

The zero mode wave functions of the left-handed electron and tau, the right-handed zero mode of ψ_1 and the first excited state of the tau are displayed in fig. 1. Since the SM neutrinos are at different locations in the extra dimension, non-hierarchical Yukawa couplings typically lead to nearest neighbor-type neutrino mass matrices. This is similar to the case of quark mixings which we discussed in ref. [5]. By taking the Yukawa coupling of ν_τ to be somewhat smaller than unity, it is however possible

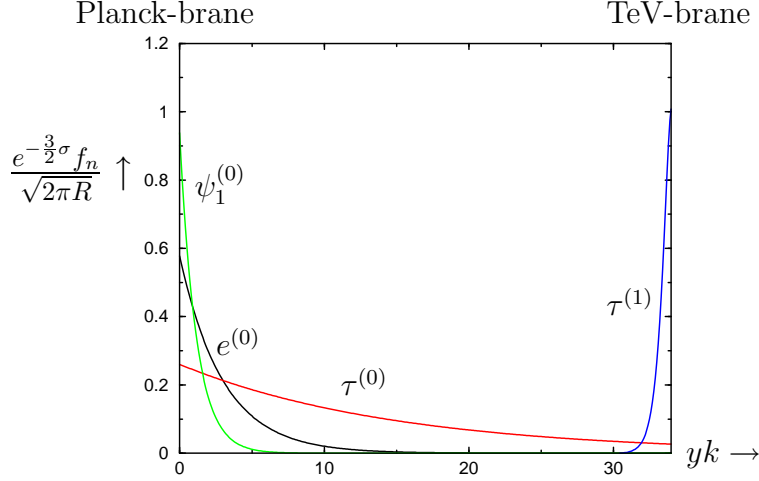


Figure 1: Localization of the electron, τ and ψ_1 zero modes, and the first KK state of the τ in the extra dimension for the parameters of eq. (4.14).

to obtain large ν_μ - ν_τ mixing. Large ν_e - ν_μ mixing would require a hierarchy of order 10^2 at least in the Yukawa couplings, which leaves the small MSW scenario as the only natural solution to the solar neutrino anomaly. The data are, for instance, reproduced by taking

$$c_\psi^{(1)} = 1.38, \quad c_\psi^{(2)} = 1.29, \quad c_\psi^{(3)} = 1.24,$$

$$\frac{h_{ij}^{(5)}}{g_2^{(5)}} = \begin{pmatrix} 1.1 & 2.9 & 3.3 \\ -2.1 & -3.6 & 3.8 \\ -0.5 & -0.4 & 0.3 \end{pmatrix}. \quad (4.14)$$

For the light neutrino masses we obtain $m_{\nu 1} = 8.0 \cdot 10^{-6}$ eV, $m_{\nu 2} = 2.6 \cdot 10^{-3}$ eV and $m_{\nu 3} = 7.6 \cdot 10^{-2}$ eV. The lightest KK state is basically a ν_τ excitation with a mass of 10.4 TeV. The excitations of the other bulk fields have similar masses. The neutrino data are successfully reproduced with $\Delta m_{\text{atm}}^2 = 5.8 \cdot 10^{-3}$ eV², $\sin^2 2\theta_{\text{atm}} = 0.92$, $\Delta m_{\text{sol}}^2 = 6.8 \cdot 10^{-6}$ eV² and $\sin^2 2\theta_{\text{sol}} = 3.0 \cdot 10^{-3}$. The quantity $U_{e3}^2 = 5 \cdot 10^{-6}$, much smaller than the experimental bound. This example demonstrates that the small angle MSW solution can be obtained from order unity parameters. This is in contrast to “neutrino mass anarchy models” in four dimensions [17].

The SM neutrinos mix not only with each other but also with the left-handed KK states of the sterile neutrinos $\psi_L^{(i)}$ [13]. This effect diminishes the effective weak charge of the light neutrinos. As a result the effective number of neutrinos contributing to the width of the Z boson is reduced to $n_{\text{eff}} = 3 - \delta n$, where δn is obtained from summing the relevant squared entries of U . Measurements of the Z width induce the constraint $\delta n \lesssim 0.005$ [18]. For the parameter set (4.14) we find the result $\delta n = 6 \cdot 10^{-7}$, well below the experimental sensitivity. Because of this small mixing with KK states, the masses and mixings of the light neutrinos can be reliably computed from the zero mode part of the neutrino mass matrix (3.12). In ref. [13] a result three orders of magnitude larger was reported. Our result is smaller

since our KK masses are ~ 10 TeV instead of 1 TeV, and being bulk fields, the SM neutrinos have a smaller overlap with the Higgs field at the TeV-brane which reduces the off-diagonal entries $m^{(0,i)}$ in (3.12).

In order to reproduce the large mixing angle solutions to the solar neutrino problem we have to relocate the left-handed lepton fields in the extra dimension. A nearest neighbor-type neutrino mass matrix is avoided by placing the SM neutrinos all at the same position, i.e. $c_L^{(1)} = c_L^{(2)} = c_L^{(3)} \equiv c_L$. We choose $c_L = 0.567$. In this case our results concerning proton decay [5] are still valid. The charged fermion masses are reproduced with $c_E^{(1)} = 0.787$, $c_E^{(2)} = 0.614$ and $c_E^{(3)} = 0.50$. A parameter set which implements the large angle MSW solution is

$$c_\psi^{(1)} = 1.43, \quad c_\psi^{(2)} = 1.36, \quad c_\psi^{(3)} = 1.30,$$

$$\frac{h_{ij}^{(5)}}{g_2^{(5)}} = \begin{pmatrix} -2.0 & 1.5 & -0.5 \\ -1.8 & -1.1 & 1.9 \\ 0.5 & 1.9 & 1.7 \end{pmatrix}. \quad (4.15)$$

We obtain the light neutrino masses $m_{\nu 1} = 1.0 \cdot 10^{-3}$ eV, $m_{\nu 2} = 1.0 \cdot 10^{-2}$ eV and $m_{\nu 3} = 7.1 \cdot 10^{-2}$ eV. The KK spectrum is similar to the small MSW case. For the neutrino oscillation parameters we find $\Delta m_{\text{atm}}^2 = 4.9 \cdot 10^{-3}$ eV², $\sin^2 2\theta_{\text{atm}} = 0.99$, $\Delta m_{\text{sol}}^2 = 1.0 \cdot 10^{-4}$ eV² and $\sin^2 2\theta_{\text{sol}} = 0.90$. Also, $U_{e3}^2 = 0.036$ is close to the experimental bound which is typical for the solutions we find. Since the SM neutrinos now are closer to the TeV-brane, their mixing with the sterile neutrinos is enhanced. We find $\delta n = 2 \cdot 10^{-5}$, still well below the experimental bound.

Along the same lines it is also possible to reproduce the LOW and the vacuum solution to the solar neutrino anomaly. A smaller Δm_{sol}^2 is obtained by moving the sterile neutrinos ψ_1 and ψ_2 closer towards the Planck-brane. We find viable solutions with $c_\psi^{(1)} = 1.50$ and $c_\psi^{(1)} = 1.45$ (LOW solution), and $c_\psi^{(1)} = 1.62$ and $c_\psi^{(1)} = 1.57$ (vacuum solution). The mixing between SM neutrinos and sterile neutrinos is of the size found in the large MSW solution.

5 Rare processes

The direct experimental signature for bulk SM fields would be the production of the associated KK states in high energy colliders. Since typical KK masses in our scenario are of order 10 TeV, this is not possible before the advent of LHC. It is therefore important to study the implications of the excited states on rare processes and precision variables.

In order to prevent large Majorana neutrino masses, we imposed lepton number. As a result the proton is stabilized as well. However, baryon number may still be violated by $\Delta B = 2$ operators contributing to neutron anti-neutron oscillations and double nucleon decay [15, 16]. To be specific let us consider the 6-fermion operator $O_{\Delta B=2} = U_1 D_1 D_2 U_1 D_1 D_2$. The right-handed quark fields U_i and D_i are assumed

to be bulk fields. Experiments constrain the suppression scale of this operator by $M_{\Delta B=2} \gtrsim 3.3 \cdot 10^5$ GeV [16]. In ref. [5] we discussed how the quark masses and mixings can be obtained by locating them at different positions in the extra dimension. Using the 5d quark mass parameters from that analysis, $c_{D1} = 0.57$, $c_{U1} = 0.63$, $c_{D2} = 0.57$, we estimate $M_{\Delta B=2} \sim 8 \cdot 10^5$ GeV. The computation of $M_{\Delta B=2}$ is a straight forward generalization of the 4-fermion operator case discussed in refs. [4, 5], where the effective 4d operator is obtained by integrating the fermion wave functions over the extra dimension. Since in 4 dimensions $O_{\Delta B=2}$ is suppressed by five powers of $M_{\Delta B=2}$, the rate of $\Delta B = 2$ processes is about 10^{-2} below the experimental bound. However, the 5d mass parameters are not uniquely fixed by their masses and mixings. If we move the quark fields closer to the TeV-brane, e.g. $c_{D1} = 0.54$, $c_{U1} = 0.60$, $c_{D2} = 0.54$, $M_{\Delta B=2}$ comes down to the experimental limit. The quark masses and mixings are recovered by reducing the quark Yukawa coupling by a common factor of about 5 compared to the values given in ref. [5]. Flavor changing operators are still sufficiently suppressed by a scale of about 10^6 GeV.

Non-zero neutrino masses violate the lepton flavor symmetry, which induces processes like $\mu \rightarrow e\gamma$. The rate for these transitions is considerably enhanced by the presence of heavy neutrino states [19, 20]

$$\Gamma(\mu \rightarrow e\gamma) \propto \left| \sum_i U_{ei}^* U_{\mu i} \mathcal{F}\left(\frac{m_i^2}{M_W^2}\right) \right|^2. \quad (5.16)$$

The matrix elements U_{ei} and $U_{\mu i}$ encode the admixture of the i th mass eigenstate in ν_e and ν_μ , respectively. If all neutrino masses are much smaller than M_W , i.e. $\mathcal{F}(m_i^2/M_W^2) \sim \mathcal{F}(0)$, the rate (5.16) is suppressed due to the unitarity of U . Heavy neutrinos prevent this cancellation. Lepton flavor violation in the warped geometry model with TeV-brane SM neutrinos and bulk sterile neutrinos [13] was studied in ref. [14]. It was found that the branching ratio for $\mu \rightarrow e\gamma$ exceeds the experimental limit, $\text{Br}(\mu \rightarrow e\gamma) < 1.2 \cdot 10^{-11}$ [21], if the first KK state of the sterile neutrino has a mass below 25 TeV. Thus, lepton flavor violation imposes the most stringent constraint on the model.

Evaluating the branching ratio from (5.16) in our scenario with bulk SM fermions leads to a somewhat different conclusion. The rate for $\mu \rightarrow e\gamma$ is very sensitive to the mixing between light and heavy neutrino states. With bulk neutrinos the mixing with heavy states is considerably reduced, as was discussed in the previous section. Therefore, we expect that the constraints from lepton flavor violation should be less stringent in our scenario. Indeed, for the example implementing the small MSW solution (4.14), we find $\text{Br}(\mu \rightarrow e\gamma) \sim 10^{-26}$, which is far below the experimental sensitivity. In the case of the large MSW solution (4.14) the result is considerably enhanced because of the larger mixing of SM and sterile neutrinos. We obtain $\text{Br}(\mu \rightarrow e\gamma) \sim 10^{-15}$, still well below the experimental sensitivity. The branching ratios for $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ are also much too small to be observed in present

experiments. For the LOW and vacuum solutions to the solar neutrino anomaly, the rate for lepton flavor violation is similar to the result for the large MSW example.

In principle, the rates for lepton flavor violation can be considerably enhanced by moving the SM neutrinos (and with them the charged left-handed leptons) closer to the TeV-brane. For instance, if we take $c_L = 0.5$ as the common 5d mass parameter of the left-handed leptons, we can obtain $\text{Br}(\mu \rightarrow e\gamma) \sim 10^{-13}$ with the large MSW solution. The current experimental sensitivity is almost reached for $c_L = 0.45$ which leads to $\text{Br}(\mu \rightarrow e\gamma) \sim 5 \cdot 10^{-12}$. In order to reproduce the charged lepton and neutrino masses, the right-handed charged leptons and sterile neutrinos have to reside closer to the Planck-brane. However, for $c_L < 0.5$ the effective 4d weak gauge of the left-handed leptons start to deviate from the gauge couplings of the right-handed leptons that live close to the Planck-brane, since the W and Z boson wave functions are not strictly constant in the extra dimension [7]. The associated violation of lepton universality renders this part of the parameter space less attractive. For $c_L > 0.5$, lepton flavor violation may still be only a few orders of magnitude below the present experimental limit.

6 Conclusions

We have shown how atmospheric and solar neutrino oscillations, especially the bi-maximal mixing scenario, can be incorporated in models with a small extra dimension and non-factorizable (warped) geometry. An important distinction from earlier work [13] is the appearance of the SM fermions in our case as bulk fields. Rare processes such as $n - \bar{n}$ oscillations and $\mu \rightarrow e + \gamma$ may occur at a rate not too far below the current experimental limits.

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